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STRUCTURE OF ELECTROHYDRODYNAMIC SHOCK WAVES IN MULTICOMPONENT AND MULTIPHASE MIXTURES FOR THE CASE OF ARBITRARY INTERACTION PARAMETER

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The structure of shock waves is investigated for the case of multicomponent and multiphase mixtures containing a neutral gas and ($N - 1$) grades of charged particles; each grade is characterized by its own mobility coefficient. The interaction parameter is arbitrary. The analysis is made for the case of small Prandtl numbers when the temperature of the medium can be considered constant and for the case of large Prandtl numbers when heat conducting processes can be neglected. As in conventional electrohydrodynamics [1], the solution of the structure problem for an electrohydrodynamic shock wave depends substantially on the velocity direction, the electric field and on the current density ahead of the wave front. For definiteness, everywhere throughout the following analysis it is assumed that the velocity normal component $u_{\mathrm{I}} > 0$ ahead of the shock wave front. It is shown, that if the electric field ahead of the wave has a component normal to the front of the shock wave $E_{\mathrm{I}} > 0$, there is always a shock wave structure and the electric field at the wave front is continuous. If $E_{\mathrm{I}} < 0$ and the current densities of all components are positive, a shock wave structure does not occur for all values of parameters ahead of the wave front. If the structure exists, then it follows from the analysis that the electric field at the wave front is either continuous or discontinuous. In the latter case the electric field and velocity components normal to the wave front are related behind the wave front by the equation $u_{\mathrm{II}} + b_2 E_{\mathrm{II}} = 0$, where b_2 is the greatest mobility coefficient of the mixture components. The surface charge at the shock wave front is caused by a sharp increase in the charge density of the mixture component having the greatest mobility coefficient. As follows from the shock wave analysis, in the case when $E_{\mathrm{I}} < 0$ and current densities of all components are negative, the electric field at the shock wave front is continuous, if ahead of the wave front the sum $u_{\mathrm{I}} + b_N E_{\mathrm{I}} \neq 0$ (b_N is the smallest mobility coefficient of the components). If the sum $u_{\mathrm{I}} + b_N E_{\mathrm{I}} = 0$, the field at the wave front may suffer a discontinuity. In order to determine the parameters behind the wave front, one of the parameters behind the wave must be specified. If, moreover, behind the wave front $u_{\mathrm{II}} + b_N E_{\mathrm{II}} \neq 0$, then the velocity ahead of the wave is greater and behind it smaller than the speed of sound. If $u_{\mathrm{II}} + b_N E_{\mathrm{II}} = 0$ then the gas velocity ahead of and behind the shock wave front is supersonic. The existence of this kind of shock wave structure was pointed out in conventional electrohydrodynamics [1] for one grade of charged particles. When the motion of a multicomponent medium is considered, the component with the smallest mobility coefficient acts as that with one grade of charged particles. When $E_1 < 0$ and the current densities of the components have different signs, the shock wave structure does not exist for all parameter values ahead of the wave front. Let the current densities j_2,

$j_3, \ldots, j_p$ be negative and $j_{p+1}, j_{p+2}, \ldots, j_N$ – positive. The form of additional relations which close the system of equations of the shock wave front can be found by analyzing the wave structure. Moreover, there are some cases when the electric field at the shock wave front is either continuous or discontinuous and the parameters behind the wave front or ahead of the wave, are expressed by the relations $u_{II} + b_{p+1}E_{II} = 0$ or $u_I + b_pE_I = 0$, respectively. In the latter case one of the parameters behind the wave front must be specified in order to determine the remaining parameters. Moreover, if $u_{II}+b_pE_{II}\neq 0$, the velocity ahead of the shock wave front is supersonic and behind it – subsonic. There exists a shock wave structure in which ahead of the wave front the sum $u_I+b_pE_I=0$ and behind the front $u_{II}+b_pE_{II}=0$, and the gas velocity ahead of, and behind the shock wave front is supersonic. From the analysis of the shock wave structure, if the structure exists, it follows that in the case of a negative field $(E_I < 0)$ and currents of different signs, there exists a type of shock wave for which, ahead of the front and behind it, the parameters of the medium satisfy the equations $u_I + b_pE_I = 0$ and $u_{II} + b_{p+1}E_{II} = 0$. The gas velocity ahead of the wave is supersonic and behind the wave – subsonic. The density distribution of the total bulk charge inside the structure of such waves has two maxima. The first maximum is caused by an increase in the charge density of p-grade of charged particles (changes in charge densities of other components are small), the second maximum – by an increase in the charge density of $(p+1)$ grade of charged particles, while there is only a small change in the charge density of the remaining components. Shock waves of this type can be formed only in mixtures containing charged particles of different grade; they do not exist in conventional electrohydrodynamics.

1. Let us consider (applying the electrohydrodynamic approximation) a stationary flow of a multicomponent or multiphase medium – of a gas and $(N-1)$ grade of ions or charged particles. We shall denote by a subscript $\alpha\,(\alpha = 2, 3, \ldots, N)$ parameters relating to the corresponding grade of charged particles. We direct the x coordinate axis along the stream velocity and assume that the electric field has a single component parallel to the axis. Let all parameters of flow be dependent only on x. To define a flow of charged particles or drops, if those are present, we use the diffusion approximation [2]. Equations defining this flow allowing for viscosity and heat conduction (see also [1–3]) can be written as

$$\rho u = 1, \quad \frac{du}{d\zeta} = u + \frac{1}{\gamma M_0^2}\frac{T}{u} - SE^2 - \Pi, \quad \Pi = \text{const} \tag{1.1}$$

$$\frac{3}{4(\gamma-1)M_0^2\,\mathrm{Pr}}\frac{dT}{d\zeta} + u\frac{du}{d\zeta} = \frac{1}{(\gamma-1)M_0^2}T + \frac{1}{2}u^2 + 2SJ(\varphi-\varphi_0) - \Sigma \tag{1.2}$$

$$p = \rho T, \qquad \Sigma = \text{const}$$

$$\frac{dE}{d\zeta} = \sum_{\alpha=2}^{N}\frac{\varepsilon_\alpha}{E+R_{q\alpha}u}, \quad \frac{d\varphi}{d\zeta} = -QE, \quad R_{q\alpha}J_\alpha = q_\alpha(E+R_{q\alpha}u) \tag{1.3}$$

$$J_\alpha = \text{const}, \quad \alpha = 2, \ldots, N$$

The dimensionless parameters from Eqs. (1.1)–(1.3) are given by formulas

$$\rho=\frac{\rho^*}{\rho_0^*},\quad u=\frac{u^*}{u_0^*},\quad p=\frac{p^*}{p_0^*},\quad T=\frac{T^*}{T_0^*} \tag{1.4}$$

$$E=\frac{E^*}{E_0^*},\quad q_\alpha=\frac{q_\alpha^*}{q_0^*},\quad q_0^*=\sum_{\alpha=2}^{N}\overset{*}{q}_{\alpha 0},\quad \varphi=\frac{4\pi q_0^*\varphi^*}{E_0^{*2}}$$

$$M_0=\frac{u_0^*}{a_0^*},\quad a_0^{*2}=\frac{\gamma p_0^*}{\rho_0^*},\quad \gamma=\frac{c_p}{c_v},\quad S=\frac{E_0^{*2}}{8\pi\rho_0^* u_0^{*2}}$$

$$Q=\frac{4\pi q_0^* l}{E_0^*},\quad J_\alpha=\frac{j_\alpha^*}{q_0^* u_0^*},\quad R_{q\alpha}=\frac{u_0^*}{b_\alpha E_0^*}$$

$$Pr=\frac{c_p\eta}{\varkappa},\quad J=\sum_{\alpha=2}^{N}J_\alpha,\quad \varepsilon_\alpha=R_{q\alpha}QJ_\alpha=\frac{4\pi l j_\alpha^*}{b_\alpha E_0^{*2}}$$

$$\zeta=\frac{x}{l},\qquad l=\frac{4\eta}{3\rho_0^* u_0^*}$$

Here ρ^*, u^*, p^*, T^* are the dimensional density, velocity, pressure and temperature of the medium, respectively; $q_\alpha^*>0$, j_α^*, E^*, φ^* are, respectively, the dimensional bulk charge density, the current density, the electric field intensity and the electric potential, b_α is the mobility coefficient of charged particles of α-grade; η, $\varkappa$ are the coefficients of viscosity and thermal conductivity of the mixture (in the subsequent analysis the values of b_α, η, $\varkappa$ are considered constant); c_p, c_v are the specific heats, l is the length of the mean free path.

By the zero subscript we denote the parameters of a particular point of the stream at which the terms appearing in the left-hand sides of the second of Eqs. (1.1) and of the first of Eqs. (1.2),and related to viscosity and thermal conductivity, can be neglected. The integration constants Π and Σ defined by flow parameters at this point are

$$\Pi=1+\frac{1}{\gamma M_0^2}-S,\qquad \Sigma=\frac{1}{(\gamma-1)M_0^2}+\frac{1}{2} \tag{1.5}$$

According to the law of conservation and Maxwell's equations, the system of relations at the shock wave front in a multiphase medium is not closed, similarly as in conventional electrohydrodynamics [1, 3]. The missing relation in conventional electrohydrodynamics was obtained in [3] for the case of small and in [1] for the case of an arbitrary interaction parameter. In the electrohydrodynamics of multiphase media corresponding relations for a small interaction parameter were derived in [2, 4],and for the case of a large interaction parameter and small Prandtl numbers in [5]. In the latter case it can be assumed that the temperature of the medium is constant and the first equation of (1.2) replaced by $T=1$.

Below, the structure of a shock wave is analyzed and the equations are found which close the system of relations at the shock wave front in a multicomponent or multiphase medium for the case of an interaction parameter for any quantity of components when Prandtl number $\mathrm{Pr}\gg 1$, which corresponds to a low thermal conductivity of the mixture.

Neglecting in the first equation of (1.2) the term with the derivative $dT/d\zeta$, assuming $\varphi=\varphi_0$ [1] and using the second equation of (1.1), we obtain the expression for the gas temperature

$$T = \gamma(\gamma - 1) M_0^2 (^1/_2 u^2 - SE^2 u - \Pi u + \Sigma) \tag{1.6}$$

Let us examine the behavior of the integral curves from Eqs. (1.1), (1.3) and (1.6) in the semi-plane uE, $u > 0$. We divide the first equation of (1.3) by the second equation of (1.1) in which the temperature T is expressed by (1.6), and we obtain

$$dE/du = \sum_{\alpha=2}^{N} \varepsilon_\alpha \prod_{\beta\neq\alpha} (E + R_{q\beta} u) \Big[L_{(2)} \prod_{\alpha=2}^{N} L_{(1)\alpha} \Big]^{-1} \tag{1.7}$$

$$L_{(1)\alpha} = E + R_{q\alpha} u, \quad L_{(2)} = \frac{\gamma+1}{2} u + \left(\frac{1}{M_0^2} + \frac{\gamma - 1}{2} \right) \frac{1}{u} - \gamma SE^2 - \gamma \left(1 + \frac{1}{\gamma M_0^2} - S \right)$$

Assuming $q_\alpha^* > 0$, the value

$$|\varepsilon_\alpha| = 4\pi q_0^* l |R_{q\alpha} J_\alpha| \, |E_0^*| \ll 1$$

and coefficients $b_2 > b_3 > \ldots > b_N$ in the all region of flow, and $|R_{qN}| > |R_{q(N-1)}| > \ldots > |R_{q_2}|$. Let at the beginning $E_0^* < 0$. The lines $L_{(1)\alpha} = 0$ and $L_{(2)} = 0$ are the isoclines on which the integral curves of Eq. (1.7) have vertical tangents ($dE/du = \infty$). These lines, the behavior of which is determined by parameters γ, M_0, $R_{q\alpha}$ and S, we denote by $\overset{\circ}{L}_{(1)\alpha}$ and $L_{(2)}^\circ$, respectively. The form of the line $L_{(2)}^\circ$ was examined in [1]. In Figs. 1–5 the case is shown, when in the plane uE the line $L_{(2)}^\circ$ for $u > 0$, $E > 0$ has one branch with an extremum at the point u_m, E_m. All results are easily generalized to the case of a different position of $L_{(2)}^\circ$. The form of the line $L_{(3)}^\circ$, on which the velocity is equal to the speed of sound, was also investigated in [1]. The intersection of this line with $L_{(2)}^\circ$ is only possible at extrema of the line $L_{(2)}^\circ$ (if they exist). If $L_{(2)}^\circ$ and $L_{(3)}^\circ$ intersect, then the parts of $L_{(2)}^\circ$ lying in the interval $u < u_m$ are in the subsonic region and those in the interval $u > u_m$ are in the supersonic region; the velocity at the point of intersection is sonic. One of the possible behaviors of the line $L_{(3)}^\circ$ is shown in Fig. 1 by a dashed line. In the other figures this line is not shown in order to have more readable graphs.

Applying reasoning analogous to that given in [1], we obtain that lines $\overset{\circ}{L}_{(1)\alpha}$ and $L_{(2)}^\circ$ can intersect either at one point lying in the subsonic or supersonic region or at three points – Figs. 1–5. In the latter case all points may be situated in the supersonic region – Figs. 2, 4, 5; or two of them are in the supersonic and one in the subsonic region – Figs. 1, 3. The coordinates of the points of intersection are determined by the following equations:

$$E = -R_{q\alpha} u$$

$$SR_{q\alpha}^2 u^3 - \frac{\gamma+1}{2\gamma} u^2 + \left(1 + \frac{1}{\gamma M_0^2} - S\right) - \left(\frac{1}{\gamma M_0^2} + \frac{\gamma-1}{2\gamma}\right) = 0$$

Then we shall examine the behavior of the integral curves of Eqs. (1.7) only in those regions where they have a physical meaning. Integral curves which run below the isocline $\overset{\circ}{L}_{(1)2}$ between the isoclines $\overset{\circ}{L}_{(1)p}$ and $\overset{\circ}{L}_{(1)p+1}$ and those above the isocline $\overset{\circ}{L}_{(1)N}$, define flows for $j_\alpha > 0$ (Fig. 1); for $j_p < 0$, $j_{p+1} > 0$ (Figs. 3–5) and for $j_\alpha < 0$ (Figs. 1, 2), respectively (everywhere $\alpha = 2, 3, \ldots N$, $2 \leqslant p \leqslant (N-1)$). In order not to complicate the graphs, only two from $(N-1)$ $\overset{\circ}{L}_{(1)\alpha}$ isoclines are shown, namely: $\overset{\circ}{L}_{(1)2}$ and $\overset{\circ}{L}_{(1)N}$; they correspond to the components with the greatest and the smallest mobility coefficients in Fig. 1,and $\overset{\circ}{L}_{(1)p}$, $\overset{\circ}{L}_{(1)p+1}$ in Figs. 3–5.

2. Let us consider a mixture flow when $j_\alpha > 0,\ \alpha = 2, \ldots, N$. Integral curves defining such a flow lie below the isocline $\overset{\circ}{L}_{(1)2}$(Fig. 1). The direction of motion along the integral curves is shown by arrows in Fig. 1. An investigation of the shock waves shows that the form of the relations obtained for the wave structure depends substantially on the position of the isoclines $L_{(2)}{}^\circ$ and $\overset{\circ}{L}_{(1)2}$. We shall consider the most interesting case when the lines $L_{(2)}{}^\circ$ and $\overset{\circ}{L}_{(1)2}$ intersect at three points A_2, B_2 and C_2 (the point C_2 is not shown in Fig. 1); the point A_2 is in the subsonic, while B_2 and C_2 are in the supersonic region. We denote the electric field value of these points by E_{a2}, E_{b2}, E_{c2}. We consider that these parts of the integral curves in the super- and subsonic regions, where integral curves run in the ε-neighborhood of $L_{(2)}{}^\circ$, and represent a gas flow ahead of and behind the shock wave, respectively. Points at which integral curves (for $\varepsilon \to 0$) leave the ε-neighborhood of the isocline $L_{(2)}{}^\circ$, correspond to the states ahead of the shock wave. Point at which, for $\varepsilon \to 0$, integral curves enter the ε-neighborhood of the isocline $L_{(2)}{}^\circ$, correspond to the states behind the shock wave. Parts of integral curves which are parallel to the axis $E = 0$ and to the isocline $\overset{\circ}{L}_{(1)2}$, describe the structure of the electrohydrodynamic shock wave. We denote by numerals I and II the field values ahead of and behind the shock wave, respectively. By analysis analogous to that in [1], it can be shown that the integral curves with a field $E_{\mathrm{I}} \leqslant E_{a2}$, ahead of the wave front, define the shock wave structure with a continuous field at the wave front. If $E_{a2} < E_{\mathrm{I}} < E_{b2}$, then the integral curves on which the selected value of a field ahead of the wave front lies, describe a shock wave structure with a discontinuous electric field and the formation of a surface charge at the wave front. To find the value of the surface charge at the wave front, i.e. to close the system of relations at the front of the shock wave, it is necessary to use the following equations:

$$u_{\mathrm{II}}{}^* + b_2 E_{\mathrm{II}}{}^* = 0, \qquad 4\pi\sigma = -u_{\mathrm{II}}{}^* / b_2 - E_{\mathrm{I}}{}^* \tag{2.1}$$

The surface charge at the wave front is caused by a sharp increase in the charge density of the component with the highest mobility coefficient. Integral curves leaving the isocline $L_{(2)}{}^\circ$ above the point C_2 and intersecting the line $L_{(2)}{}^\circ$, do not run along it afterwards – there is no region corresponding to an inviscid flow. Such integral curves do not define a shock wave structure. It is noted, that algebraic relations following from the law of conservation at the front of the shock wave permit a jump from a state corresponding to the part described of the isocline $L_{(2)}{}^\circ$ (inviscid flow) to the state determined by the part of the isocline $L_{(2)}{}^\circ$ lying in the subsonic region (also defining an inviscid flow). Everything said above indicates that the shock waves of such type do not possess a structure.

In the case when the isoclines $\overset{\circ}{L}_{(1)2}$, $L_{(2)}{}^\circ$ intersect only at the point A_2 of the subsonic region, the field at the shock wave front is continuous and the surface charge is equal to zero, if $E_{\mathrm{I}} \leqslant E_{a2}$. If $E_{\mathrm{I}} > E_{a2}$, then integral curves on which the selected value of an electric field lies, describe a shock wave structure with a discontinuity of the electric field and the formation of a surface charge at the wave front. To determine the value of this surface charge, i.e. to close the system of relations at the shock wave front, it is necessary to use Eqs. (2.1).

In the case when the isoclines $\overset{\circ}{L}_{(1)2}$ and $L_{(2)}{}^\circ$ intersect at points in the supersonic region, there are no integral curves defining the shock wave structure.

3. Let us consider a mixture flow when $j_\alpha < 0$, $\alpha = 2 \dots N$. The isocline $\overset{\circ}{L}_{(1)N}$ corresponds to a component with the smallest mobility coefficient. Integral curves defining such a flow lie above the isocline $\overset{\circ}{L}_{(1)N}$ (Fig. 1, 2). The form of the relations obtained from the examination of the shock wave structure depends substantially on the position of the isoclines $\overset{\circ}{L}_{(1)N}$ and $L_{(2)}{}^{\circ}$. It follows from the third equation of (1.3) that $|R_{q\alpha}| < 1$, $\alpha = 2, \dots, N$. The lines $L_{(2)}{}^{\circ}$ and $\overset{\circ}{L}_{(1)N}$ may have either three points or one point of intersection. In the latter case the point of intersection lies in the supersonic region.

In Fig. 1 we have the case when the isoclines $\overset{\circ}{L}_{(1)N}$ and $L_{(2)}{}^{\circ}$ intersect at three points A_N, B_N, C_N; the point A_N lies in the subsonic region and point B_N and C_N in the supersonic region of the plane uE. We denote the values of the electric field at the points A_N, B_N and C_N by E_{aN}, E_{bN} and E_{cN}, respectively ($E_{aN} < E_{bN} < E_{cN}$). We assume that behind the wave front the electric field intensity satisfies the inequality

$$E_{aN} < E_{\mathrm{II}} < E_{bN} \tag{3.1}$$

Integral curves on which the selected value of the electric field E_{II} can lie, define a shock wave structure with a discontinuous electric field at the wave front [1]. Parameters ahead of the wave front satisfy the relation

$$u_{\mathrm{I}}^* + b_N E_{\mathrm{I}}^* = 0 \qquad (4\pi\sigma = E_{\mathrm{II}}^* + u_{\mathrm{I}}^* / b_N) \tag{3.2}$$

The electric field behind such a discontinuity can have any values satisfying inequalities (3.1). To determine flow parameters behind the shock wave front when the parameters ahead are known and the condition (3.2) is fulfilled, one of the parameters behind the wave fromt must be specified. We note that the value of the electric field behind the wave front should be within the limits determined by the inequality (3.1). The existence of this kind of shock waves in conventional electrohydrodynamics in the case of a small interaction parameter was indicated in [6]. If the electric field behind the shock wave front satisfies the inequality $E_{bN} \leqslant E_{\mathrm{II}} < E_{cN}$, then there is a continuous electric field at the front of the shock wave and the surface charge intensity is equal zero.

Let us examine some examples of flows in the case when the isoclines $L_{(2)}{}^{\circ}$ and $\overset{\circ}{L}_{(1)N}$ intersect in the supersonic region. We denote the intersection points by A_N, B_N, C_N in the case of three points, and by A_N in the case of one point of intersection. It is not difficult to see that in the case of three intersection points of the isoclines $\overset{\circ}{L}_{(1)N}$ and $L_{(2)}{}^{\circ}$ (Fig. 2) lying in the supersonic region, the electric field at the shock wave front is continuous, if behind or ahead of the wave front the inequlities $E_{bN} \leqslant E_{\mathrm{II}} < E_{cN}$, $E_{\mathrm{I}} < E_{aN}$ are satisfied, respectively. The field undergoes a discontinuity, if behind the wave front the inequalities $E_{aN} \leqslant E_{\mathrm{II}} < E_{bN}$ are satisfied; parameters ahead of the discontinuity front satisfy Eqs. (3.2). To determine the flow parameters behind the shock wave front when parameters ahead of it are related by (3.2), one of the parameters behind the wave front must be specified. In addition, the value of the electric field behind the wave front must be held within the limits defined by the inequalities $E_{aN} \leqslant E_{\mathrm{II}} < E_{bN}$. The gas velocity is subsonic in all the cases mentioned above. There exists a set of integral curves leaving the small neighborhood of the point B_N with a slope equal to the slope of the line $\overset{\circ}{L}_{(1)N}$. Below the point B_N such integral curves run along the isocline $\overset{\circ}{L}_{(1)N}$ to the point A_N and then along the isocline $\overset{\circ}{L}_{(2)}$. (In Fig. 2 the corresponding integral curve is shown as a dashed line). Such integral curves define a structure of shock waves, parameters of which ahead of and behind the wave front are related by the equations $u_l^* + b_N E_l^* = 0, \quad l = \mathrm{I}, \mathrm{II}$; the velocity behind the front of wave (as well as ahead of the wave) is supersonic and equal to the larger root of the equation $L_{(2)}(u, E_{aN}) = 0$.We note that there exists a shock wave structure for which the velocity behind the wave front is subsonic and equal to the smaller root of the equation $L_{(2)}(u, E_{aN}) = 0$; the field behind the wave front $E_{\mathrm{II}} = E_{aN}$, while ahead of the front it satisfies the relation (3.2).

Let the isoclines $\overset{\circ}{L}_{(2)}$, $\overset{\circ}{L}_{(1)N}$ have only a single point of intersection lying in the supersonic region. We assume that the field ahead of the wave front satisfies the inequality $E_{\mathrm{I}} < E_{aN}$. Integral curves on which the selected value of the electric field can lie, define the structure of the shock wave with a continuous field. Other shock waves having structure do not exist in this case.

4. Let us consider the flow when $j_\alpha < 0$ for $\alpha = 2, 3, \ldots, p$ and $j_\alpha > 0$ for $\alpha = p+1, \ldots, N$. In Fig. 3 the isoclines $L_{(2)}{}^\circ$, $\overset{\circ}{L}_{(1)p}$ and $\overset{\circ}{L}_{(1)p+1}$ are constructed. Integral curves defining such a flow are disposed between the isoclines $\overset{\circ}{L}_{(1)p+1}$ and $\overset{\circ}{L}_{(1)p}$. Investigation of the shock wave structure shows that the form of relations for this structure depends substantially on the position of the isoclines $\overset{\circ}{L}_{(1)p+1}$, $\overset{\circ}{L}_{(1)p}$ and $\overset{\circ}{L}_{(2)}$. It follows from the third equation of (1.3) that $|R_{q\alpha}| < 1$ for $\alpha = 2, 3, \ldots, p$ and $|R_{q\alpha}| > 1$ for $\alpha = p+1, p+2, \ldots, N$. The isoclines $L_{(2)}{}^\circ$ and $\overset{\circ}{L}_{(1)p}$, as well as $L_{(2)}{}^\circ$ and $\overset{\circ}{L}_{(1)p+1}$ may have either three points of intersection, two of which or all three are in the supersonic region, or a single intersection point; the latter can lie in either the sub- or supersonic region.

4.1. Let the isocline $L_{(2)}{}^\circ$ have three points of intersection with the isoclines $\overset{\circ}{L}_{(1)p}$ and $\overset{\circ}{L}_{(1)p+1}$: A_p, B_p, C_p and $A_{p+1}, B_{p+1}, C_{p+1}$, respectively. We denote the value of the electric field at points of intersection A_p, B_p, C_p and $A_{p+1}, B_{p+1}, C_{p+1}$ by E_{ap}, E_{bp}, E_{cp} and $E_{ap+1}, E_{bp+1}, E_{cp+1}$, respectively. In Fig. 3 points A_p and A_{p+1} lie in the subsonic region; moreover, the inequality $E_{ap+1} \leqslant E_{bp}$ is satisfied.

We denote by the numeral I the integral curve which for $\varepsilon \to 0$ emerges with zero slope from the small neighborhood of the point B_p, and by the numeral II – the integral curve which enters the small neighborhood of the point A_{p+1} with zero slope (in Fig. 3 shown by dashed lines). Integral curves leaving the neighborhood of line $L_{(2)}{}^\circ$ with zero slope between the points B_p and B_{p+1} (the field ahead of the shock wave front satisfies the inequality $E_{bp} \leqslant E_{\mathrm{I}} < E_{bp+1}$) run to the isocline $\overset{\circ}{L}_{(1)p+1}$, then turn, go along the isocline and intersect the line $L_{(2)}{}^\circ$ in the neighborhood of the point A_{p+1}. These integral curves define the structure of a shock wave with a jump of the electric field at the wave front. To close the system of equations at the shock wave front it is necessary to use the conditions

$$u_{\mathrm{II}}^* + b_{p+1} E_{\mathrm{II}}^* = 0, \qquad 4\pi\sigma = -u_{\mathrm{II}}^* / b_{p+1} - E_{\mathrm{I}}^* \tag{4.1}$$

Below the integral curve I there are integral curves defining the structure of the shock waves with a jump of the electric field at the wave front, and which ahead of the wave front are characterized by

$$u_{\mathrm{I}}^* + b_p E_{\mathrm{I}}^* = 0 \quad (E_{\mathrm{I}} = -R_{qp} u_{\mathrm{I}}) \tag{4.2}$$

Integral curves below the line II intersect the isocline $L_{(2)}^{\circ}$ between points A_p and A_{p+1}. To find the state behind the front of such shock waves, one of the parameters behind the wave front must be specified. In partucular, the value of the electric field can be specified within the limits $E_{ap} < E_{II} < E_{ap+1}$.

There exists a set of integral curves defining the structure of shock waves for which, inside the structure, the density distribution of the total bulk charge has two maxima. These integral curves lie between the lines I and II. They all emerge from the neighborhood of the point B_p, run along the isocline $\overset{\circ}{L}_{(1)p}$, move from it, run with a small angle of inclination to the isocline $\overset{\circ}{L}_{(1)p+1}$, turn and enter its ε-neighborhood to intersect the isocline $L_{(2)}^{\circ}$ in the ε-neighborhood of the point A_{p+1}. One such integral curve, curve III, is shown in Fig. 3. Ahead of the front of shock waves represented by the integral curves defined above, the sum $E_I + R_{qp}u_I = 0$, and behind the front the sum $E_{II} + R_{qp+1}u_{II} = 0$. Ahead of the front of such a shock wave the velocity is supersonic and behind it – subsonic. Shock waves of this type can be found only in the media containing several grades of charged particles and cannot exist in conventional electrohydrodynamics. The first maximum in the density distribution of a bulk charge inside the wave structure is caused by an increase in charge density q_p (charge densities q_i, $i \neq p$, undergo small changes), the second maximum is caused by an increase in charge density q_{p+1} (charge densities q_i, $i \neq p+1$, change insignificantly).

If the field ahead of the wave front satisfies the inequality $E_{cp+1} \leqslant E_I \leqslant E_{cp}$, there is no structure of shock waves.

We shall now consider the case when $E_{ap+1} > E_{bp}$. We assume that the field ahead of the shock wave front satisfies the inequality $E_{ap+1} < E_I < E_{bp+1}$. Then at the shock wave front the electric field suffers a discontinuity. The parameters behind the wave front satisfy the first relation of (4.1). Let the field behind the shock wave front satisfy the inequalities $E_{ap} < E_{II} < E_{bp}$, then ahead of the wave front the condition (4.2) is fulfilled and the surface charge is formed at the wave front. In the case when one of the following inequalities is satisfied:

$$E_{bp} < E_I \leqslant E_{ap+1}, \quad E_{bp} \leqslant E_{II} < E_{ap+1}$$

the field at the wave front is continuous and the surface charge intensity ie equal zero.

Let us consider the case when the isoclines $\overset{\circ}{L}_{(1)p+1}$ and $L_{(2)}^{\circ}$ have one point of intersection A_{p+1} in the subsonic region, while the isoclines $\overset{\circ}{L}_{(1)p}$ and $L_{(2)}^{\circ}$ have three points of intersection A_p, B_p, C_p and one of them lies in the subsonic region. In this case in order to find the surface charge, the results of Sect. 4.1 can be used replacing E_{bp+1} by E_{cp} in the inequalities $E_{bp} \leqslant E_I < E_{bp+1}$, $E_{ap+1} < E_I < E_{bp+1}$

4.2. Let us consider the case when all three points of intersection of the isoclines $L_{(2)}^{\circ}$ and $\overset{\circ}{L}_{(1)p}$ are in the supersonic region and one of the points of intersection of the isoclines $L_{(2)}^{\circ}$ and $\overset{\circ}{L}_{(1)p+1}$ lies in the subsonic region; moreover, we assume $E_{ap+1} \leqslant E_{bp}$ and $E_{ap} < E_{ap+1}$ (Fig. 4). By numerals I and III we denote integral curves which for $\varepsilon \to 0$ leave with zero slope the small neighborhood of points B_p and A_p, respectively, and by the numeral II – integral curve which enters the small neighborhood of the point A_{p+1} with zero slope. Integral curves emerging, for $\varepsilon \to 0$, with zero slope from the neighborhood of the line $L_{(2)}^{\circ}$ between points B_p and B_{p+1} (the field ahead of the shock wave front satisfies the inequality $E_{bp} < E_I < E_{bp+1}$), run with zero slope to the isocline $\overset{\circ}{L}_{(1)p+1}$, then along it and intersect the isocline $L_{(2)}^{\circ}$ in the neighborhood of the point A_{p+1}. To close the system of equations at the shock wave front and to calculate the bulk charge intensity, Eqs. (4.1) are used.

Below the integral curve III lie integral curves which define the structure of shock waves with a continuous electric field at the wave front $(E_m < E_{\text{I}} < E_{ap})$.

Between the lines I and III there are integral curves defining the structure of shock waves in which parameters ahead of the wave front are related by formulas (4.2). All these integral curves emerge from the neighborhood of the point B_p and first run along the isocline $\overset{\circ}{L}_{(1)p}$. We note that the integral curves lying between lines I and II define a shock wave structure in which the parameters of the medium ahead of the wave front satisfy the equation $u_{\text{I}} + b_p E_{\text{I}} = 0$ and behind the front – the relation $u_{\text{II}} + b_{p+1}E_{\text{II}} = 0$. Ahead of the shock wave front the velocity is supersonic and behind the front – subsonic. Inside the structure of such shock waves the bulk charge density distribution has two maxima. The first maximum of the density distribution in the bulk charge is caused by an increase in the charge density q_p (changes in charge densities q_i, $i \neq p$ are small), the second maximum is caused by an increase in charge density q_{p+1} (changes in charge densities q_i, $i \neq p + 1$, are small). Shock waves of this type are possible only in the electrohydrodynamics of a multiphase medium and are impossible in the electrohydrodynamics of charges particles of a single grade. Among the integral curves emerging from the small neighborhood of the point B_p with an inclination equal to that of the isocline $\overset{\circ}{L}_{(1)p}$, there is a set of integral curves which run along the isocline $\overset{\circ}{L}_{(1)p}$ to the point A_p and then along the isocline $L_{(2)}{}^{\circ}$. Such integral curves define a shock wave structure with parameters ahead of and behind the wave front which are related by the equation $u_l{}^* + b_p E_l{}^* = 0$, $l = \text{I}, \text{II}$; the velocity behind the wave front (as well as ahead of the wave) is supersonic and equal to the larger root of the equation $L_{(2)}(u, E_{ap}) = 0$. We note that there exists a structure of the wave for which the velocity behind the wave front is subsonic and equal to the smaller root of the equation $L_{(2)}(u, E_{ap}) = 0$; the field behind the wave front $E_{\text{II}} = E_{ap}$, and ahead of the wave satisfies the relation (4.2). Integral curves, situated between lines II and III define a shock wave structure in which parameters ahead of the front are related by (4.2). To close the system of relations at discontinuities, the structure of which is defined by such integral curves, it is necessary to specify one of the parameters behind the wave front; in this case the field must be specified within the interval $E_{ap} < E_{\text{II}} < E_{ap+1}$. If ahead of the wave front $E_{cp+1} \leqslant E_{\text{I}} \leqslant E_{cp}$, there is no shock wave structure.

In the case when $E_{ap} \geqslant E_{ap+1}$, there is an analogous method of analysis. When $E_{bp} < E_{\text{I}} < E_{bp+1}$, $E_{ap+1} \leqslant E_{\text{I}} < E_{ap}$, the parameters behind the wave front are related by (4.1). When $E_m < E_{\text{I}} \leqslant E_{ap+1}$, the field behind the wave front is continuous, etc.

We shall now consider the case when $E_{ap+1} > E_{bp}$. Let the field ahead of the wave front satisfy the inequalities $E_{ap+1} < E_{\text{I}} < E_{bp+1}$. Then at the shock wave front the electric field suffers a discontinuity and the parameters behind the front satisfy the first relation of (4.1). If behind the wave front the field satisfies the inequality $E_{ap} \leqslant E_{\text{II}} < E_{bp}$, the parameters ahead of the front are related by (4.2) and a surface charge is formed at the front. In the case when one of the following inequalities is fulfilled

$$E_{bp} < E_{\text{I}} \leqslant E_{ap+1},\ E_{bp} \leqslant E_{\text{II}} < E_{ap+1},\ E_{\text{I}} < E_{ap}$$

the field at the wave front is continuous and the surface charge intensity is equal to zero.

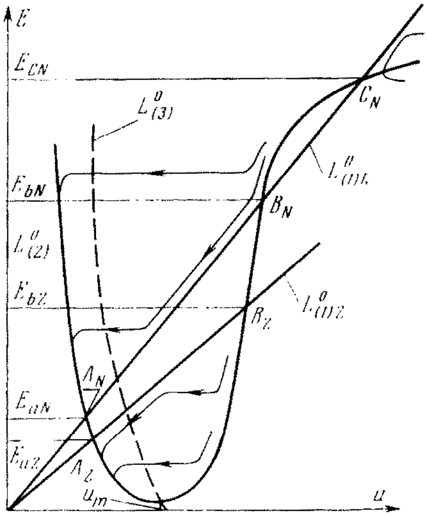

Fig. 1

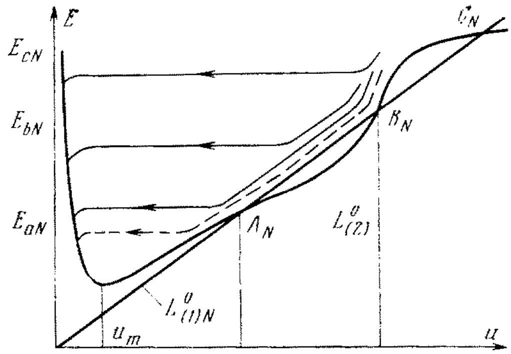

Fig. 2

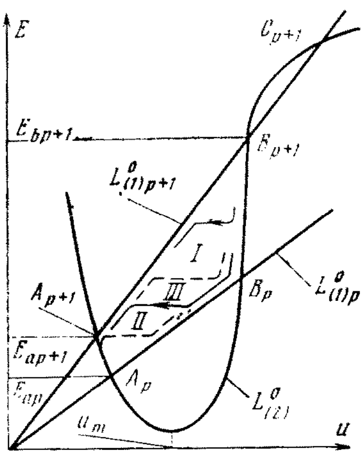

Fig. 3

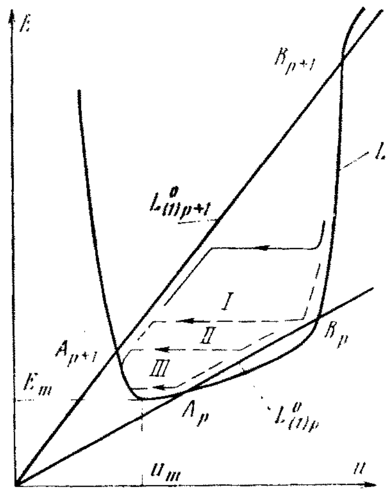

Fig. 4.

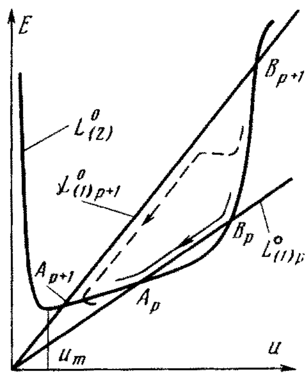

Fig. 5

There exists a shock wave structure for which, ahead of and behind the wave front, the relations $E_l^* + b_p u_l^* = 0$, $l = \mathrm{I}, \mathrm{II}$ are fulfilled. The velocity behind the wave front (as well as ahead of the wave) is supersonic and equal to the larger root of the equation $L_{(2)}(u, E_{ap}) = 0$. We note that there exists a wave structure for which the velocity behind the wave front is subsonic and equal to the smaller root of the equation $L_{(2)}(u, E_{ap}) = 0$. The field behind the wave front $E_{\mathrm{II}} = E_{ap}$ and ahead of the wave front, satisfies the relation (4.2). If $E_{cp+1} \leqslant E_{\mathrm{I}} \leqslant E_{cp}$, a shock wave structure does not exist.

Let the isoclines $\overset{\circ}{L}_{(2)}$ and $\overset{\circ}{L}_{(1)p}$ have three points of intersection in the supersonic region, while isoclines $L_{(2)}{}^{\circ}$ and $\overset{\circ}{L}_{(1)p+1}$ have a single point of intersection A_{p+1} lying in the subsonic region. In this case the surface charge can be determined using the results of Sect. 4.2 and replacing E_{bp+1} by E_{cp} in the inequalities $E_{bp} < E_{\mathrm{I}} < E_{bp+1}$, $E_{ap+1} < E_{\mathrm{I}} < E_{bp+1}$.

Now we consider the case when the isoclines $L_{(2)}{}^{\circ}$ and $\overset{\circ}{L}_{(1)p}$ intersect at a single point A_p in the supersonic region, and the isoclines $\overset{\circ}{L}_{(2)}$ and $\overset{\circ}{L}_{(1)p+1}$ intersect in the subsonic region at the point A_{p+1}. The position of the isoclines differs from that in Fig.4: the supersonic branch of the isocline $L_{(2)}{}^{\circ}$ intersects the isocline $\overset{\circ}{L}_{(1)p}$ at the point A_p and then continues to run below it. Let us assume that ahead of the wave front the field satisfies the inequalities $E_{\mathrm{I}} < E_{ap}$, if $E_{ap} \leqslant E_{ap+1}$ or $E_{\mathrm{I}} \leqslant E_{ap+1}$, if $E_{ap} > E_{ap+1}$. The integral curves, on which a selected value of the electric field can lie, define a shock wave structure with a continuous field at the wave front. Let us assume that the field ahead of the wave front satisfies the inequalities $E_{ap+1} < E_{\mathrm{I}} < E_{ap}$, when $E_{ap} > E_{ap+1}$. The integral curves, on which a selected value of the electric field can lie, define a shock wave structure with a discontinuity of the electric field. The field behind the wave front must satisfy the first relation of (4.1). In this case there are no other shock waves possessing structures.

4.3. Let us consider the case when the isocline $L_{(2)}{}^{\circ}$ has three points of intersection A_p, B_p, C_p and $A_{p+1}, B_{p+1}, C_{p+1}$ with each of the isoclines $\overset{\circ}{L}_{(1)p}$ and $\overset{\circ}{L}_{(1)p+1}$, respectively, and all three intersection points lie in the supersonic region (Fig. 5). Among the integral curves leaving the small neighborhood of point B_p with an inclination equal to that of the isocline $\overset{\circ}{L}_{(1)p}$, there are integral curves which run along the isocline $\overset{\circ}{L}_{(1)p}$ to the point A_p and then along the isocline $\overset{\circ}{L}_{(2)}$. Such integral curves define a shock wave structure with parameters ahead of and behind the wave front related by $u_l^* + b_p E_l^* = 0$, $l = \mathrm{I}, \mathrm{II}$. The gas velocity behind the wave front (as well as ahead of the wave) is supersonic and equal to the larger root of the equation $L_{(2)}(u, E_{ap}) = 0$. There are no other integral curves defining a shock wave structure. In fact, integral curves which intersect the isocline $L_{(2)}{}^{\circ}$ in the neighborhood of the point A_{p+1}, do not run along the isocline at all. In other words, an inviscid flow ahead of the shock wave, corresponding to the segment $B_p B_{p+1}$ of the isocline $L_{(2)}{}^{\circ}$, cannot be linked with any inviscid flow related to the state behind the shock wave.

Analogous conclusions can be reached when the isoclines $\overset{\circ}{L}_{(2)}$ and $\overset{\circ}{L}_{(1)p+1}$ have a single point of intersection A_{p+1} in the supersonic region and the isoclines $\overset{\circ}{L}_{(2)}$ and $\overset{\circ}{L}_{(1)p}$ intersect at three points A_p, B_p, C_p lying in the supersonic region. In this case the shock wave structure exists, and its velocity behind and ahead of the front is supersonic.

In the case when the isocline $\overset{\circ}{L}_{(2)}$ intersects the isocline $\overset{\circ}{L}_{(1)p+1}$ at a single supersonic point and has a single point of intersection with the isocline $\overset{\circ}{L}_{(1)p}$ lying in the supersonic region, there are no integral curves defining the shock wave structure.

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TWO-DIMENSIONAL AND AXISYMMETRIC NONSTATIONARY GAS FLOWS WITH STRONG SHOCK WAVES

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We propose a way of obtaining the second and successive approximations in constructing a solution by Chernyi's method [1–3]. Limiting nonstationary flows of an inviscid gas were studied in [4, 5].

1. We consider the self-similar motion of a gas behind a strong shock wave propagating according to the law

$$x = N_0 t f_1 (s), \quad y = N_0 t f_2 (s) \tag{1.1}$$

Here x and y are Cartesian coordinates, N_0 is a characteristic velocity of displacement of the shock wave front, and t is time. If we let s denote arc length of the wave front in the plane of self-similar variables, then the functions f_1 and f_2 must satisfy the condition $f_1'^2 + f_2'^2 = 1$. In the axisymmetric case we take the x, y plane to be a meridional plane with the x-axis as the axis of symmetry.

We assume that all the hydrodynamic characteristics of the flow depend on two variables ξ and η. We write the gasdynamic equations in Lagrangian variables; by virtue of self-similarity, these variables are introduced in the form